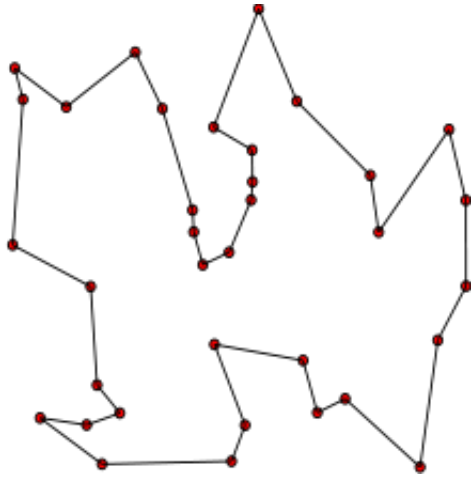
Vehicle Routing Problem

Decision-aid Methodologies in Transportation: Computer Lab 4

Travelling Salesman Problem



Informal

Objective: min cost/distance

Subject To:

- Each city is visited exactly once
  - Flow in
  - Flow out
- One salesman – no subtours

Mathematical

$$s.t. \qquad x_{ij} \in (0,1), \qquad \forall i \in I, \forall j \in J,$$

Mathematical

$$\begin{aligned} \textit{min} \quad & \sum_{i \in I} \sum_{j \in J : j \neq i} c_{ij} \cdot x_{ij} \\ \text{s.t.} \quad & x_{ij} \in (0,1), \qquad \forall i \in I, \forall j \in J, \end{aligned}$$

Mathematical

$$\min \quad \sum_{i \in I} \sum_{j \in J : j \neq i} c_{ij} \cdot x_{ij}$$

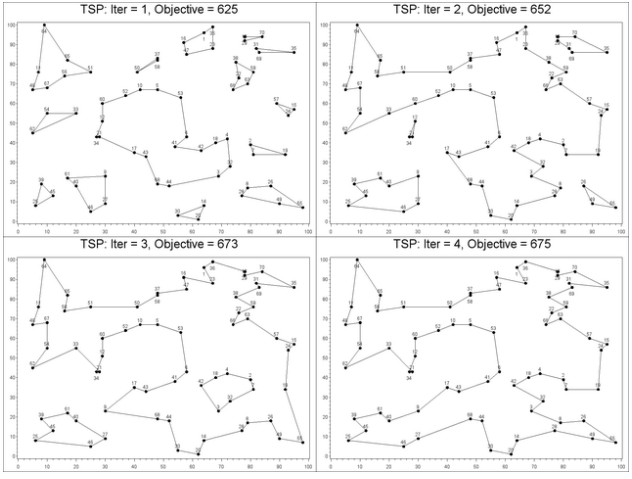
$$\text{s.t.} \quad x_{ij} \in (0,1), \qquad \forall i \in I, \forall j \in J,$$

$$\sum_{i \in I : i \neq j} x_{ij} = 1, \qquad \forall j \in J,$$

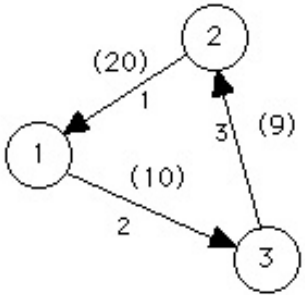
Mathematical

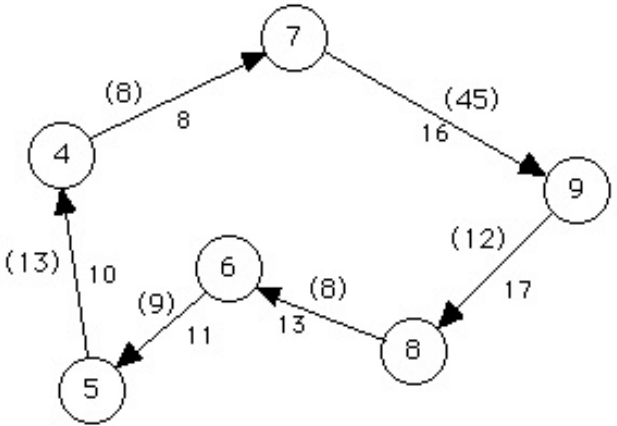
$$
\begin{aligned}
\textit{min} \quad & \sum_{i \in I} \sum_{j \in J : j \neq i} c_{ij} \cdot x_{ij} \\
\text{s.t.} \quad & x_{ij} \in (0,1), && \forall i \in I, \forall j \in J, \\
& \sum_{i \in I : i \neq j} x_{ij} = 1, && \forall j \in J, \\
& \sum_{j \in J : j \neq i} x_{ij} = 1, && \forall i \in I,
\end{aligned}
$$

What is a subtour?



What are the properties that we want?





Mathematical

$$
\begin{aligned}
\textit{min} \quad & \sum_{i \in I} \sum_{j \in J : j \neq i} c_{ij} \cdot x_{ij} \\
\text{s.t.} \quad & x_{ij} \in (0,1), && \forall i \in I, \forall j \in J, \\
& \sum_{i \in I : i \neq j} x_{ij} = 1, && \forall j \in J, \\
& \sum_{j \in J : j \neq i} x_{ij} = 1, && \forall i \in I, \\
& \sum_{i \in I} \sum_{j \in J : j \neq i} x_{ij} \leq |S| - 1, && \forall S \in \{2, ..., n\},
\end{aligned}
$$

Sequential Formulation - Miller, Tucker, Zemlin

$$u_i - u_j + n \cdot x_{ij} \leq n - 1, \qquad \forall i \in I \backslash \{0\}, \forall j \in J \backslash \{0\} : j \neq i,$$

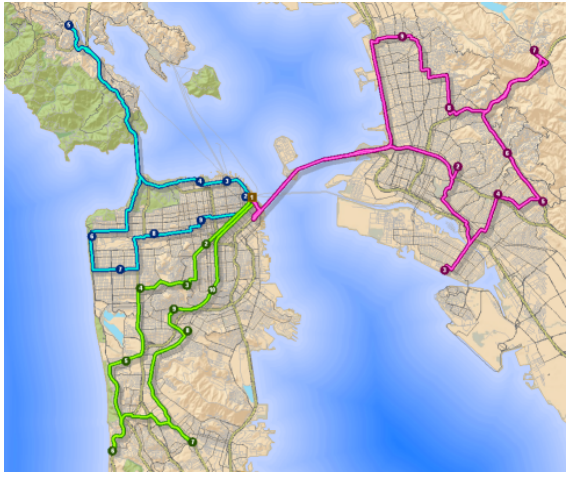
$$5 - 6 + 10 \cdot 1 \leq 9,$$
$$5 - 6 + 10 \cdot 0 \leq 9,$$
$$7 - 5 + 10 \cdot 0 \leq 9,$$
$$7 - 5 + 10 \cdot 1 \leq 9,$$
$$5 - 9 + 10 \cdot 0 \leq 9,$$
$$5 - 9 + 10 \cdot 1 \leq 9,$$

What can be solved?

24,978 Cities in Sweden
Solved in 2004

15,112 Cities in Germany
Solved in 2001

Vehicle Routing Problem



Vehicle Routing Problem

Is it related to the TSP?

- yes - each vehicle of the VRP is a TSP
- Vehicle has to visit set of customers and come back to the depot
- additional constraints per vehicle (capacity, equipment)
- additional constraints on the problem (deliver certain amount, time windows, etc.)
- MTZ subtour elimination can be adjusted according to the level